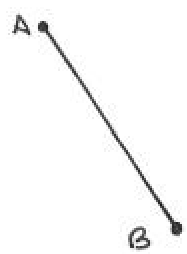
Geometry Masterpack

1. Bisecting a Segment

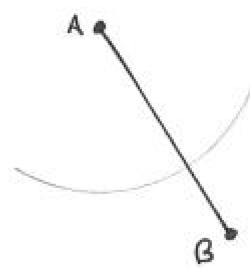
In order to bisect any segment, we start with a process.

1. Given a line segment, e.g, $\overline{AB}$, we place our compass' pointer at A, extend it so the distance will exceed half way, and create an arc.
2. *Without changing the compass' measurement*, we place its pointer now at point B and create another arc in the opposite direction. Notice the two arcs intersect! Label the points where they intersect, e.g, points C and D.
3. Now we place our straightedge on points C and D, and mark it so it now creates $\overline{CD}$. Where $\overline{AB}$ and $\overline{CD}$ intersect is now the bisector (or midpoint) of $\overline{AB}$. We can label this point, e.g, M.

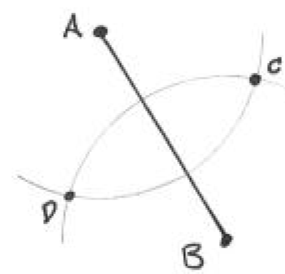
See the following images for a visual description of what these steps indicate. Afterwards, try some on your own.



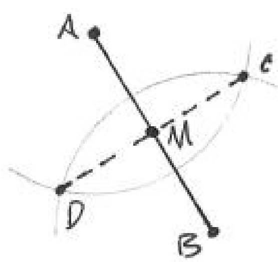
A given line segment



Step 1



Step 2



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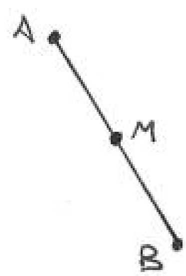


Completion

1. Bisecting a Segment

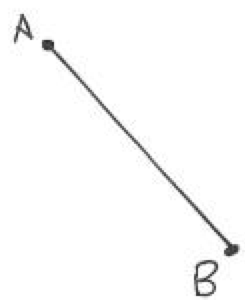
(work section)

2. Constructing a perpendicular bisector of a Segment

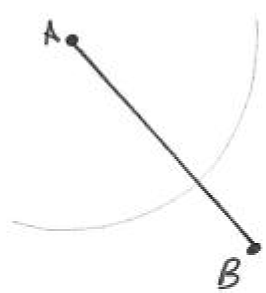
In order to construct a perpendicular bisector of any segment, we use a process.

1. Given a line segment, e.g, $\overline{AB}$, we place our compass' pointer at A, extend it so the distance will exceed half way, and create an arc.
2. *Without changing the compass' measurement*, we place its pointer now at point B and create another arc in the opposite direction. Notice the two arcs intersect! Label the points where they intersect, e.g, points C and D.
3. Now we place our straightedge on points C and D, and mark it so it now creates $\overline{CD}$. Now $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.

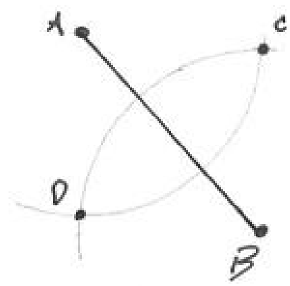
See the following images for a visual description of what these steps indicate. Afterwards, try some on your own.



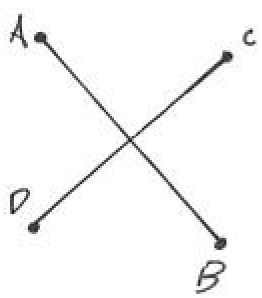
A given line segment



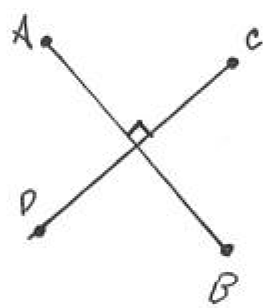
Step 1



Step 2



Step 3



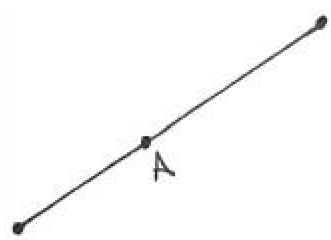
Completion

2. Constructing a perpendicular bisector of a Segment

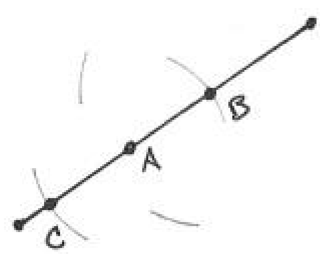
(work section)

3. Constructing a perpendicular line thru a point

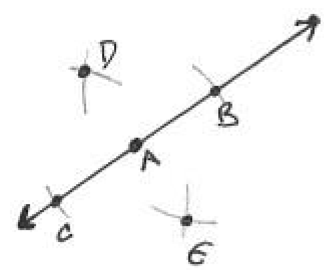
1. Given a point on a line, e.g, A, we place our compass' pointer on it. We create equidistant arcs from A, i.e, extend our compass' pencil so it will create an arc on one side of the line, and then using the same distance create an arc on the other side of the line. We can mark points where these arcs intersect the line, e.g, points B and C.
2. Place our pointer on point B and extend it past A. Now we create arcs both above and below the line; above and below A.
3. We mirror this step by repeating it thru point C *using the same distance*, creating arcs above and below A. We label these intersections, e.g, D and E.
4. Using our straightedge, we create $\overline{DE}$. Notice $\overline{DE}$ is perpendicular to the line through A.



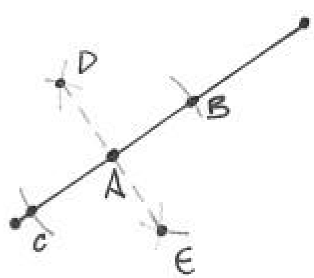
Step 1



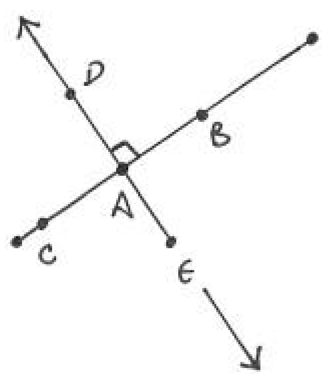
Steps 1 and 2



Step 3



Step 4



Completion

3. Constructing a perpendicular line thru a point

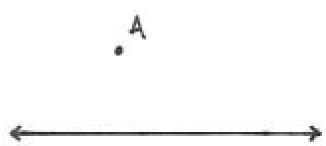
(work section)

4. Constructing a perpendicular line thru an “outsider” point

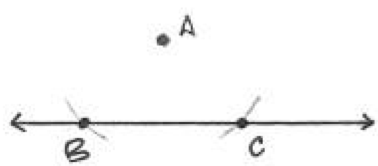
In order to construct a perpendicular line through an outsider point, we use a process.

1. Given a point not on the line, e.g, A, we place our compass’ pointer on it. We create equidistant arcs from A, i.e, extend our compass’ pencil so it will create an arc on one side of the line, and then using the same distance create an arc on the other side of the line. We can mark points where these arcs intersect the line, e.g, points B and C.
2. *Using the same distance*, we place our pointer on point B and create an arc below the line; on the opposite side of the line as A.
3. We mirror this step by repeating it thru point C *using the same distance*, creating an arc below the line; on the opposite side of the line as A. We label this intersection, e.g, D.
4. Using our straightedge, we create $\overline{AD}$. Notice $\overline{AD}$ is perpendicular to the line through A.

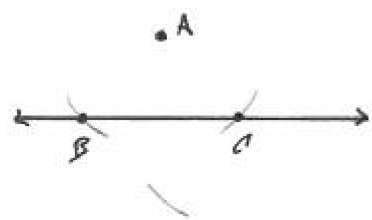
See the following images for a visual description of what these steps indicate. Afterwards, try some on your own.



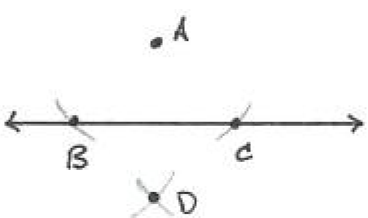
A given line with an “outsider” point



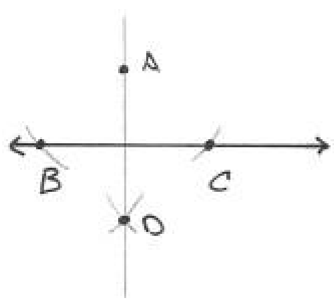
Step 1



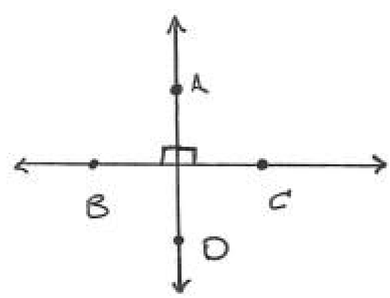
Step 2



Step 3



Step 4



Completion

4. Constructing a perpendicular line thru an “outsider” point

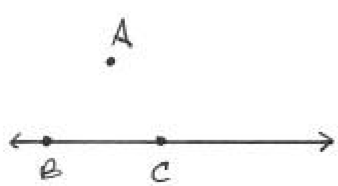
(work section)

5. Constructing parallel lines through a point not on the line

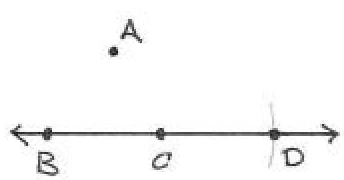
In order to construct parallel lines through an outsider point, we use a process.

1. Given a point not on the line, e.g, A, along with some given points on the line, e.g, B and C, we place our compass' pointer on C. We measure the distance from B to C and we create an arc on the other side of the line from the point we placed our pointer at (in this example, C).
2. *Using the same distance*, we place our pointer at A and create an arc across from it, as if in the direction of a line parallel to the given line.
3. We place our pointer at C and measure the distance between it and A. Using this distance, we place our compass' pointer at D and create an arc that intersects the one already created. We label this point, e.g, E.
4. Using our straightedge, we create $\overline{AE}$. Notice $\overline{AE}$ is parallel to the given line (in this example, $\overline{BC}$).

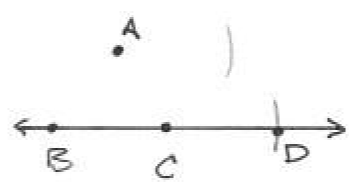
See the following images for a visual description of what these steps indicate. Afterwards, try some on your own.



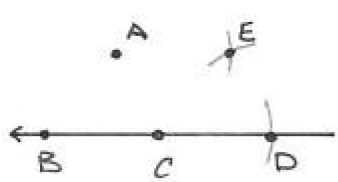
A given line with some points



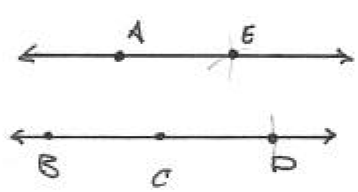
Step 1



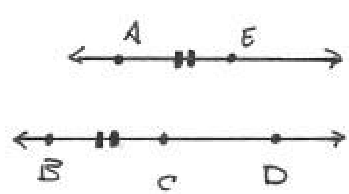
Step 2



Step 3



Step 4



Completion

5. Constructing parallel lines through a point not on the line

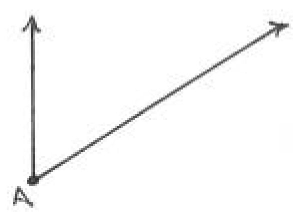
(work section)

6. Bisecting an angle

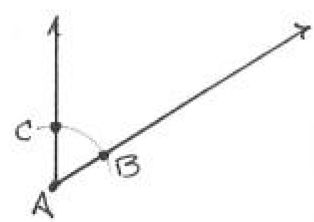
In order to construct a bisect an angle, we use a process.

1. Given an angle, e.g, A, we place our compass' pointer at A and create an arc of any size. Where this arc intersects both rays of the angle we label these points, e.g, B and C.
2. *Without changing the compass' measurement*, we place its pointer now at point B and create another arc in the interior of the angle.
3. We mirror this step by repeating it thru point C *using the same distance*, creating another arc in the interior of the angle. Where these arcs intersect is another point. We label this intersection, e.g, D.
4. Now we place our straightedge on points A and D, and mark it so it now creates $\overline{AD}$. $\overline{AD}$ is now the angle bisector of angle A.

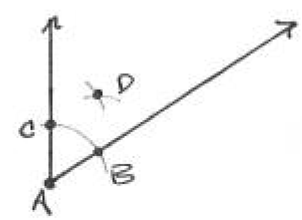
See the following images for a visual description of what these steps indicate. Afterwards, try some on your own.



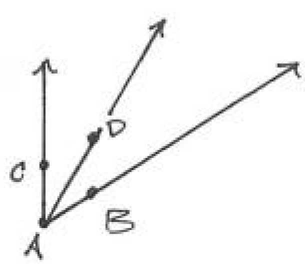
A given angle



Step 1



Steps 2 and 3



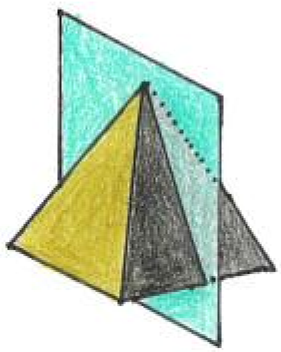
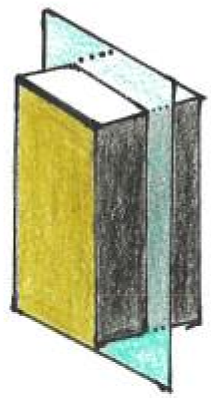
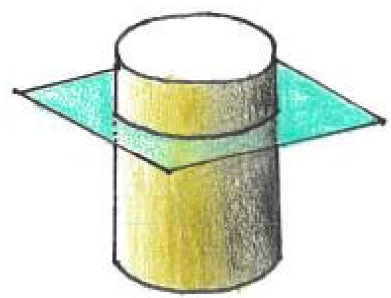
Completion

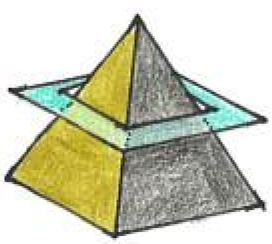
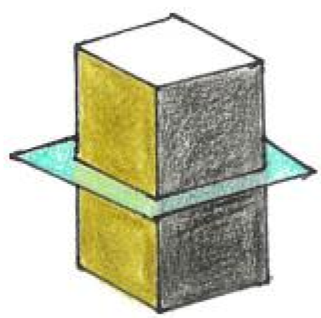
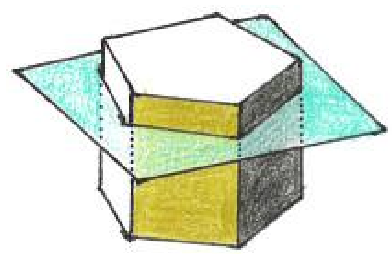
6. Bisecting an angle

(work section)

7. Cross sections of 3D solids

Identify the shapes of the cross sections of the following 3D figures.

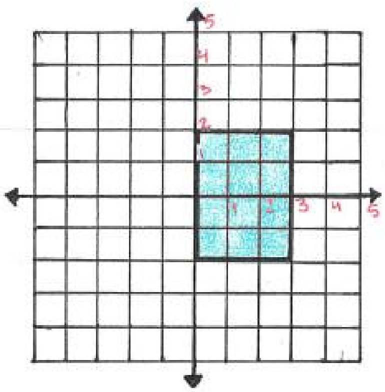




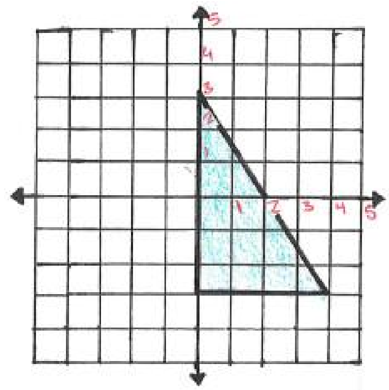
8. 3D Figures formed by rotating 2D shapes

Identify the 3D figures generated by rotating these shapes.

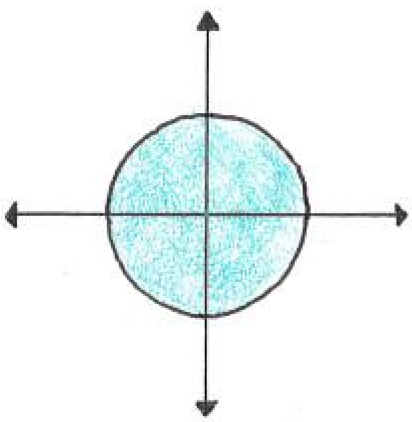
1. Rotating this rectangle about the *y*-axis.
2. Rotating this triangle about the *y*-axis.
3. Rotating this circle about the *x*-axis.
4. Rotating each of these circles about the *y*-axis.



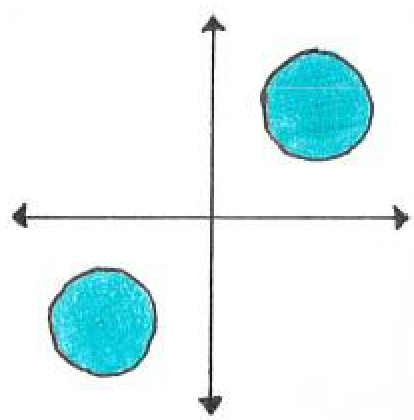
Shape 1



Shape 2



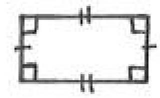
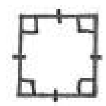
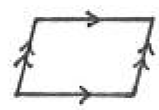
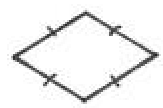
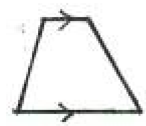
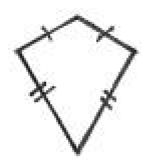
Shape 3



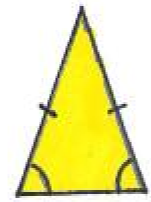
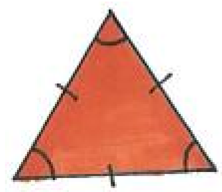
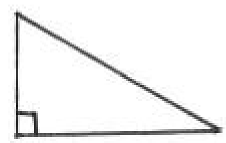
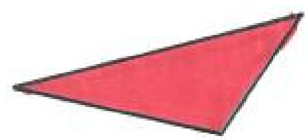
Shape 4

9. Proving / Disproving shapes from coordinates

Showing whether or not a figure on the coordinate plane is a certain shape or not requires that we know properties of different shapes. Please consult with the tables below.

Quadrilateral	*Properties*	*Diagram*
Rectangle	4 right angles and opposite sides are equal	
Square	4 right angles and 4 equal sides	
Parallelogram	Two pairs of parallel sides and opposite sides equal	
Rhombus	Parallelogram with 4 equal sides	
Trapezoid	Two sides are parallel	
Kite	Two pairs of adjacent sides of the same length	

9-a. Properties of triangles

Triangle	*Properties*	*Diagram*
Isosceles	2 sides of the same length, and 2 equal angles	
Equilateral	All sides of the same length and same angle (60°)	
Right-angled	Sides can be any length, one angle is 90°	
Scalene	All sides are different lengths, all angles are different degress	

Finding slopes and distances of figures in the plane can help us prove what shape they are.

E.g, slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

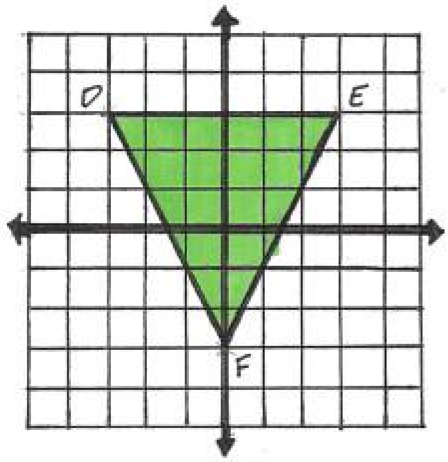
And also the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Check out the next page to see some of these techniques being used to prove if an example triangle is / is not an isoceles triangle.

9-b. An example solving shapes

Here's an example triangle:



In order to check if the example triangle (right) is an isoceles triangle or not, we should go through a process.

We know that in order for a triangle to be *isoceles*, we must have two sides of congruency (being of the same size and shape).

Let's use the distance formula to find the lengths of sides $\overline{DF}$ and $\overline{EF}$. Recall the previous section for an example of the distance formula.

Solving for length of $\overline{DF}$:

$$d = \sqrt{(0 - (-3))^2 + (-3 - 3)^2} =$$

$$d = \sqrt{9 + 36} \quad or$$

$$d = \sqrt{45}$$

Solving for length of $\overline{EF}$:

$$d = \sqrt{(0 - (3))^2 + (-3 - 3)^2} =$$

$$d = \sqrt{9 + 36} \quad or$$

$$d = \sqrt{45}$$

The lengths of these two sides are the same (both have a length of $\sqrt{45}$).

In addition (pun not intended), the length of $\overline{DE} = 6$, unequal to our other sides, which makes this triangle indeed an isoceles triangle.

Now try some on your own on the next page. Consult the previous sections, images, or even your mentor for help.

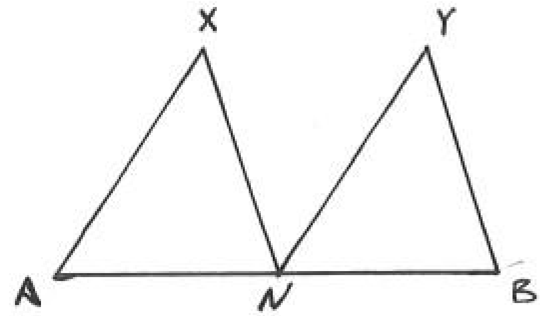
The next subsection will show some tables where we are given some statements and have to fill out the rest that are not given. Consult the previous sections, images, or even your mentor for help.

10. Other practice

1. Given that:

N is the midpoint of $\overline{AB}$
$\overline{AX}$ is congruent to $\overline{NY}$
$\overline{NX}$ is congruent to $\overline{BY}$

Prove angle X is congruent to angle Y.



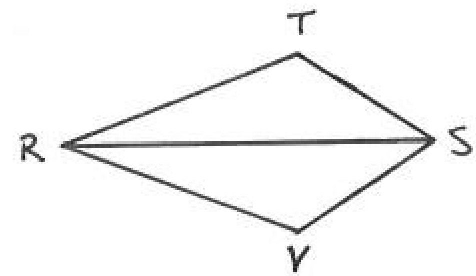
Statements	***Reasoning***
1. N is the midpoint of $\overline{AB}$	(given statement)
2.	Definition of a midpoint
3. $\overline{AX}$ is congruent to $\overline{NY}$	
4.	(given statement)
5. $\triangle AXN$ is congruent to $\triangle NYB$	
6.	

10. Other practice (cont.)

2. Given that:

$\overline{RT}$ is congruent to $\overline{RV}$
$\overline{TS}$ is congruent to $\overline{VS}$

Prove angle RST is congruent to angle RSV.



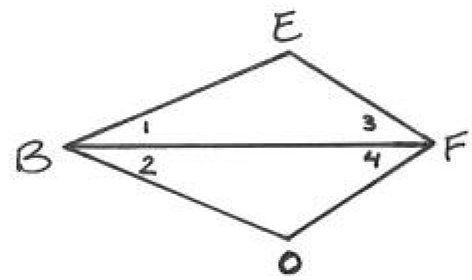
Statements	*Reasoning*
1.	(given statement)
2.	
3.	Reflexive property
4.	SSS congruency
5. Angle RST is congruent to Angle RSV	

10. Other practice (cont.)

3. Given that:

$\overline{VB}$ bisects Angle EVO & Angle EBO

Prove angle E is congruent to angle O.



Statements	***Reasoning***
1. $\overline{VB}$ bisects Angle EVO	
2.	Definition of Angle Bisector
3.	(given statement)
4. Angle 1 is congruent to Angle 2	
5. $\overline{BV}$ is congruent to $\overline{BV}$	
6.	ASA congruency
7. Angle E is congruent to Angle O	

11. Other practice

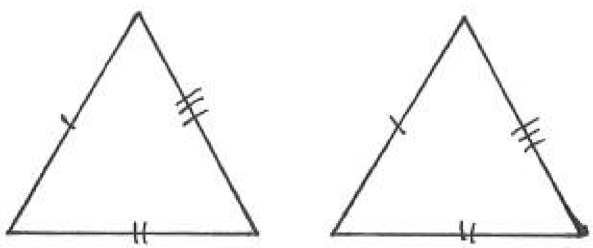
1. Prove or disprove if this shape is a square:

   $A\ (1, 2)\quad B\ (3, 2)\quad C\ (3, 0)\quad D\ (1, 0)$

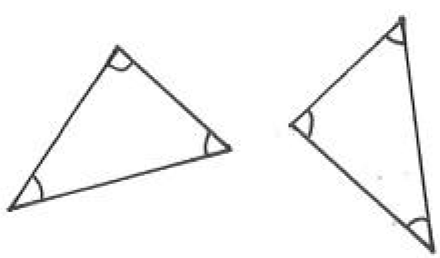
2. Prove or disprove if this shape is a parallelogram:

   $E\ (2, 8)\quad F\ (4, 4)\quad G\ (1, 1)\quad H\ (-\ 1, 5)$

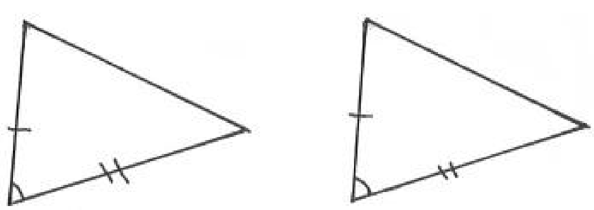
Are the following triangles congruent with their pair? If so, list the triangle congruence postulate.



Example 1

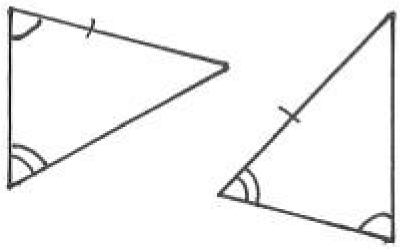


Example 2



Example 3

11. Other practice (cont.)



Example 4